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Differentiation Rules Basics:

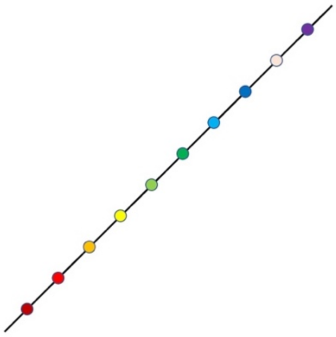
Table of Contents

These notes cover the basics of what differentiation means and how to differentiate. You will also need to learn the following differentiation applications:

- Stationary Points (maxima/minima) along with increasing/decreasing/concavity and points of inflection. See the notes 'Differentiation - stationary/turning points (max, min), points of inflection and concavity/
- Tangents and normal. See the notes 'Differentiation - Tangents and Normals'
- Optimisation. See the notes Differentiation – Optimisation'
- Differentiation - First principles -

1 What Is Differentiation?

Up until we learn differentiation, we can only find the gradient of a **straight line** i.e. the gradient between two points which is $\frac{rise}{run}$. You should be very familiar with this.



Notice how the gradient is the same at every single point on the straight line above. No matter which two points on the line that you pick, the gradient will not change. You will also be familiar with the fact that a line $y = mx + c$ has gradient m. If we pick any two points on the line (x_1, y_1) and (x_2, y_2) we can find the gradient:

Way 1 (left to right)

subtract subtract

$(x_1, y_1)(x_2, y_2)$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Way 2 (right to left)

subtract subtract

$(x_1, y_1)(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Method:

In words, this gradient formula says: subtract the y coordinates and divide by the answer we get by subtracting the x coordinates

$\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{y_2 - y_1}{x_1 - x_2}$

The formula should make sense because

$\frac{\text{rise}}{\text{run}} = \frac{\updownarrow}{\leftrightarrow}$ which is just $\frac{\text{change in } y}{\text{change in } x}$

The answer for the gradient will be the same no matter which two points on the line we pick. Gradient is also referred to as slope or rate of change (also called average rate of change).

For example, in order to find the gradient of the pink line below we can choose any points on the line (we don't choose points which aren't on a cross since we don't know their exact coordinate).

We have the following points

$(-9,7)$ $(-6,6)$ $(-3,5)$ $(0,4)$ $(3,3)$ $(6,2)$ $(9,1)$ $(12,0)$

Pick ANY pair of coordinates. Let's choose $(-3,5)$ and $(3,3)$

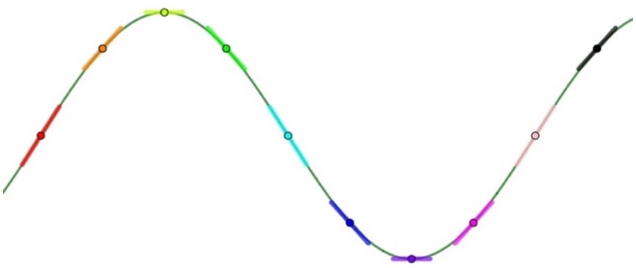
Way 1 (subtract left to right)

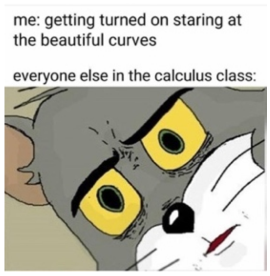
$$m = \frac{5-3}{-3-3} = \frac{2}{-6} = -\frac{1}{3}$$

Way 1 (subtract right to left)

$$m = \frac{3-5}{3--3} = \frac{-2}{6} = -\frac{1}{3}$$

What about the gradient of a **curve**? You can see from the coloured lines drawn at each of the points below (these lines are known as tangent lines) that the gradient changes at every point. The tangent lines do not have the same slope at every point – some are steeper than others and some slopes are negative and some are positive.





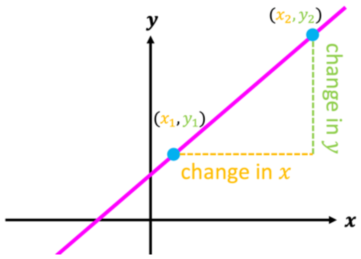
Because the gradient is now **constantly** changing at different points along the curve $f(x)$, we **no longer** want to find the average slope **between 2 points,** but **instead** the slope **at a single point** (so we need to find the slope at a point not between points which means we can no longer use the simple slope formula $\frac{y_2-y_1}{x_2-x_1}$). We can draw the coloured tangent lines as shown to approximate the gradient at different points. The slope of a curve $f(x)$ at a point is just the slope of the tangent at that point i.e. the gradient of a curve at a point is defined as the slope of a tangent line at that particular point. This means finding the slope of the tangent lines at each of the points finds us the gradient at each of the points. To find an **approximation** of the gradient at any point we can any pick any two points on the tangent line drawn at that point and calculate the gradient using our familiar formula $\frac{\text{rise}}{\text{run}}$ or $\frac{y_2-y_1}{x_2-x_1}$ since the tangent lines are straight lines. These tangent lines drawn are <u>only an approximation</u> though! We want <u>the exact slope/gradient</u> at these points. So, how can we find the exact slope at these points? Since the gradient constantly changes as seen above, we need some rule that can give the gradient at any given point. This rule is known as differentiation! **Differentiation gives us this rule in order to find the slope/gradient at any given point.**

In summary, finding the slope **between two points** above is known as finding an **average rate of change (**left diagram below), whereas finding a slope evaluated **at** a **single point** (via differentiation) is called the **instantaneous rate of change** (right diagram below) and we use differentiation to this. Derivatives are all about change. They show how fast something is changing at a point.

Rates of Change

<u>Average</u> rate of change – slope <u>between</u> 2 points

Pick any 2 points on the line

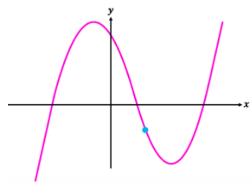


$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_1 - y_2}{x_1 - x_2}$$

Note: It should make sense that this is called average because we are <u>between</u> points.

<u>Instantaneous</u> rate of change – slope <u>at</u> a point

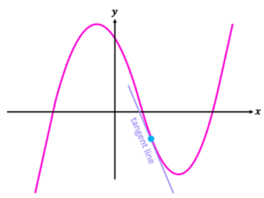
Pick any point on the curve



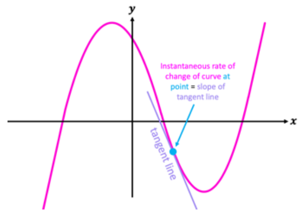
How do we find the slope at a point? There is nothing to measure now that we no longer have 2 points. There is no change in y and no change in x.

$$\text{slope} = \frac{0}{0} = ???$$

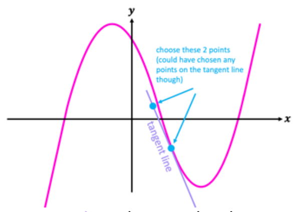
Instead, we can draw a tangent line to the curve at the point (a tangent line touches the curve at a single point and does not cross through the curve).



The tangent line will have the same slope as the curve at the point.



To find the slope so far we have always used 2 points with the formula $\frac{y_2-y_1}{x_2-x_1}$, right? Well, we could still do this by picking any 2 points on the tangent line and proceeding as on the left.



There is only one possible tangent line that can be drawn at a single point, but drawing this tangent line by hand and reading the exact coordinates of the points off of the graph isn't very accurate and only gives us an approximation. We need a more accurate method!

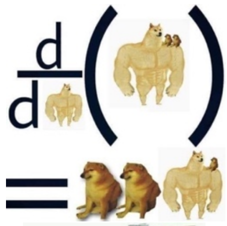
	To get an accurate exact value we must differentiate instead and plug the x value of the point in. This is also much faster! We will see how to differentiate and do this in the next chapter. Note: It should make sense that this is called instantaneous because we are AT a point.

2 Method

2.1 Basic Rule And Step By Step Method

As mentioned above, differentiation gives us a rule in order to find the slope/gradient at any given point. The rule to differentiate is:

"bring the power down to multiply with the coefficient at the front and take 1 away from the power"



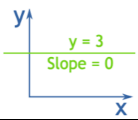
Note: You may not have an obvious coefficient at the front which is fine. When nothing is written it just means a coefficient of 1 (see example 1 below).

This is known as the **power rule** and is written formally as $x^n \rightarrow nx^{n-1}$

For example,

Example 1: x^7	**Example 2:** $4x^5$
When differentiated this becomes $7x^{7-1}$ Simplifying gives $= 7x^6$	When differentiated this becomes $(4)(5)x^{5-1}$ Simplifying gives $= 20x^4$

Be aware that:

<u>constants disappear</u>	<u>x terms just go to their coefficient in front</u>
<u>Why?</u> Consider the constant term, 3 3 means the same thing as $3x^0$ since any constant term is just the x^0 term Using the differentiation rule gives $3(0)x^{0-1} = 0$ Hence constants disappear since anything times zero is zero. Bringing that 0 power down kills everything ☺  **<u>Alternative explanation:</u>** This should also make sense from a graphical point of view. A constant, by definition never changes value and so has no gradient hence 0. 	<u>Why?</u> Consider the x term, $3x$ $3x$ means the same thing as $3x^1$ Using the differentiation rule gives $3(1)x^{1-1} = 3x^0$ Anything to the power 0 is 1 $= 3(1) = 3$ Notice how this just went to the coefficient $3x \rightarrow 3$ Hence x terms go to their coefficient **<u>Alternative explanation:</u>** This should also make sense from a graphical point of view. The slope of a **line** like $2x$ is 2, or $3x$ is 3 etc and hence it is always equal to the constant in front of x.  Note: If we differentiate a straight line $y = mx + c$ we get m which is still in keeping with the gradient of a line always being m.

2.2 Dealing With More Than One Term

What happens if we don't just have 1 term to differentiate? Let's say we have a 'string' of terms (i.e. a sum or difference) of terms like

$$2x^3 + 4x^2 - 3x + 1$$

We just deal with each term separately and apply the differentiation rule on each term one by one.

Differentiate $2x^3 + 4x^2 - 3x + 1$

Using the rule: "bring the power down to multiply with the coefficient at the front and take 1 away from the power".

$$(2)(3)x^{3-1} + 4(2)x^{2-1} - 3(1)x^{1-1} + 0$$

$$= 6x^2 + 8x - 3x^0$$

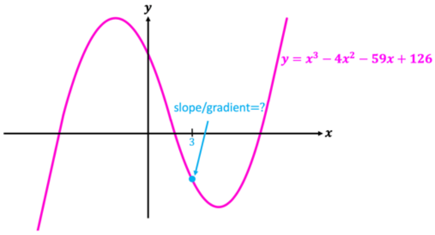
$$= 6x^2 + 8x - 3$$

2.3 Gradients At Points

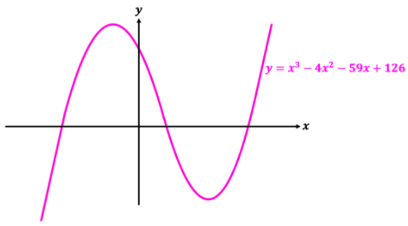
2.3.1 Given A Point, Find The Gradient

So far, by differentiating we have found a general rule that allows us to find slope/gradient at **any** point x. What about if we want to find a derivative of a curve at a **specific point**?

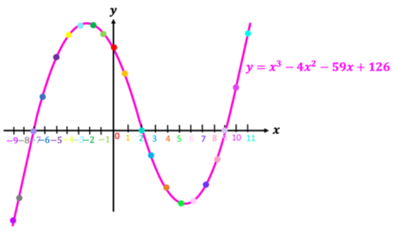
For example, we want to find the slope/gradient of the following curve $y = x^3 - 4x^2 - 59x + 126$ at a certain point $x = 3$. Let's draw a picture to help us see what is going on.



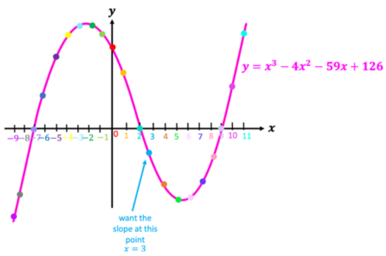
Differentiating gives us slope/gradient $= 3x^2 - 8x - 59$. This equation gives us a rule and allows to find the value of the slope/gradient at ANY point on the curve.



For example, to find the slope/gradient at any of the following points below on the curve we would just plug the x value of the point into the slope/gradient which is $3x^2 - 8x - 59$ (we could also have added in the in between values like $x = 2.5$ etc on the diagram below, but it would have complicated the diagram).



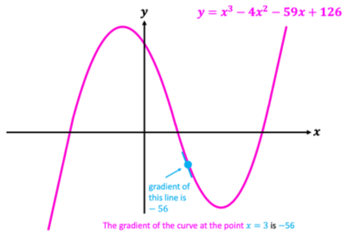
We want the slope/gradient at the point $x = 3$.



So, we plug in the point $x = 3$ into $3x^2 - 8x - 59$

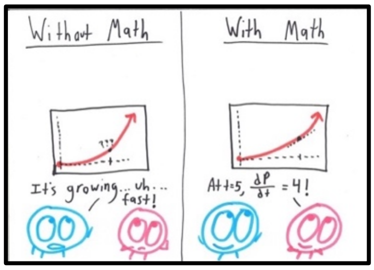
$$\text{slope/gradient} = 3(3)^2 - 8(3) - 59 = -56$$

So, the slope/gradient of the curve at the point $x = 3$ is -56 . This means any tangent line drawn at this point has gradient -56 since the gradient of the tangent line represents the gradient of the curve at the point that the tangent is drawn.



It is important that you understand this for when you start studying the topic ‘tangents and normals.’

2.3.2 Given The Gradient, Find A Point



What about if we are given the gradient and asked to find the point? For example:

Find the coordinates of the point on the graph of the function $y = 2x^2 - x + 1$ where the gradient equals 7

Our curve is

$$y = 2x^2 - x + 1$$

We differentiate to get the gradient

$$\text{gradient} = 4x - 1$$

we are told that the gradient $= 7$. This allows us to form an equation that we can work backwards from and solve for x

$$4x - 1 = 7$$

Now forget all the work we did before in order to arrive at this. Just imagine you are asked for solve $4x - 1 = 7$. This is easy right? We just use our knowledge of how to solve equations (watch out if this was a quadratic instead though i.e. an equation with a power 2 as we would have to use our knowledge of solving quadratics which means we would need to factorising or use the quadratic formula).

We now solve for x. First we add 1 to both sides.

$$4x = 8$$

Now we divide both sides by 4 to find x.

$$x = 2$$

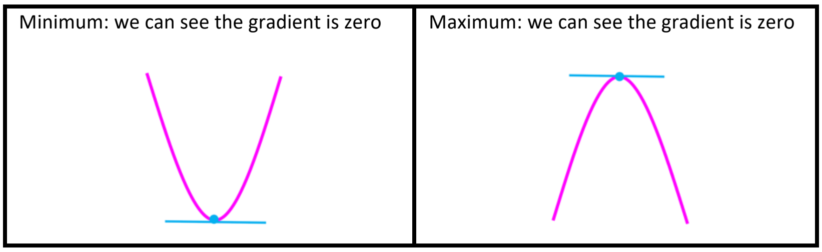
The question asked us for the coordinate, so we also need y also. We have a rule for y the curve equation) which was $y = 2x^2 - x + 1$. So, we can just plug in $x = 2$ into the curve equation.

When $x = 2$,

$$y = 2(2)^2 - 2 + 1 = 7$$

$\therefore$ coordinates of the point is $(2, 7)$.

You’ll often be ask to find maximum/minimum points which are also called stationary/turning points. They are just a special case of the gradient being 0. This looks like:



is a line with zero gradient

So, we just set the gradient equal to zero and solve. This topic is covered in detail in differentiation stationary/turning points notes which goes into detail about maximum/minimum/increasing/decreasing/points of inflection and concavity.

2.3.3 Dealing With Unknowns

Sometimes the curve equation will have an unknown variable in it (apart from x). We are given the gradient and the coordinate and can work backwards to find the unknown. For example,

$f(x) = ax^3 + 6x^2$. The point P lies on the graph of f. At P, $x = 1$. The graph of f has gradient 3 at the point P. Find the value of a.

The curve is

$$f(x) = ax^3 + 6x^2$$

This has an extra unknown a

Differentiating gives

$$\text{gradient} = 3ax^2 + 12x$$

We are given that the gradient is 3 when $x = 1$. This means we can substitute in $x = 1$ and set the gradient equal to 3

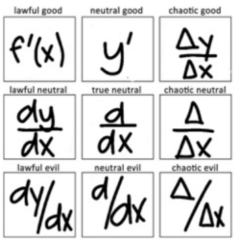
$$3a(1)^2 + 12(1) = 3$$

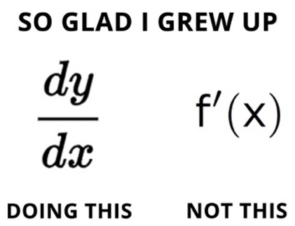
$$3a + 12 = 3$$

We now have an equation to solve

$$a = -3$$

3 Notation





We now know how to differentiate, but this is not enough. You won't always be told in words to differentiate or to find the derivative of. The derivative (also called differentiation) can be **written/represented** in several ways. This can cause some confusion when we first learn about differentiation and that is why this document covered the method and reason behind differentiation first. Now that you know what this topic is about it should be easy to take in and understand the notation.

Recall your first encounter of functions $f(x)$. You should have been taught that y means the same thing as $f(x)$ so $y = f(x)$. This means we can either be given a function as y or we can be given a function as $f(x)$.

- If we **differentiate** a function $\boldsymbol{y}$, we write y' or $\frac{dy}{dx}$ to mean we have differentiated
  Note: We are not always given y in terms of x though. We might be given different letters to y and x. For example,
  $$m = 2p^2 + 5p - 3$$
  In this case we don't write $\frac{dy}{dx}$ since we are using m and p now. The notation for differentiation basically says $\frac{\text{d(variable on the left)}}{\text{d(variable of the right)}}$
  $$\frac{dm}{dp} = 4p + 5$$
- If we **differentiate** a function $\boldsymbol{f(x)}$, we use a dash ' and write $\boldsymbol{f'(x)}$ to mean we have differentiated. This might seem a little strange at first, but you will get used to it. We pronounce it as "f - dash of x".

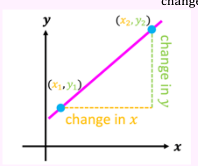
The above are equivalent ways of writing the first derivative (they just mean that we have differentiate once)

If we differentiate a function y twice (meaning again) we write $\frac{d^2y}{dx^2}$. We call this the second derivative.
If we differentiate a function $f(x)$ twice (meaning again) we write 2 dashes and hence $f''(x)$. We call this the second derivative.

For example,

Consider $2x^3 + 4x^2 - 3x + 1$

The question will not always simply say differentiate $2x^3 + 4x^2 - 3x + 1$. You could be asked this any of the 4 following ways in a more "encryped way" let's say. Way 1 and 2 are the most common, but you should also be aware of way 3 and 4.

Way 1	Way 2	Way 3	Way 4
$\boldsymbol{y = 2x^3 + 4x^2 - 3x + 1}$ **Find** $\frac{dy}{dx}$ $\frac{dy}{dx}$ just means differentiate the function y with respect to letter x. We can loosely describe this as differentiate the function y which is in terms of the letter x. Recall that the gradient of a straight line is $\frac{\text{change in y}}{\text{change in x}}$  $\frac{dy}{dx}$ just means the rate of change of y with respect to x which should make sense. $\frac{dy}{dx} = 6x^2 + 8x - 3$ If instead we had $m = 2t^3 + 4t^2 - 3t + 1$, we would have written $\frac{dm}{dt}$	$\boldsymbol{f(x) = 2x^3 + 4x^2 - 3x + 1}$ **Find** $\boldsymbol{f'(x)}$ $f'(x) = 6x^2 + 8x - 3$ This says the derivative of $f(x)$ equals $6x^2 + 8x - 3$ or simply f-dash of x equals $6x^2 + 8x - 3$ If instead we had $f(t) = 2t^3 + 4t^2 - 3t + 1$, we would have written $f'(t)$	$\boldsymbol{\frac{d}{dx}(2x^3 + 4x^2 - 3x + 10)}$ Think of $\frac{dy}{dx}$ as a noun. It is what you get after taking the derivative of y. $\frac{d}{dx}$ is a verb, it says "take the derivative of what I'm about to write after this". $\frac{d}{dx}$ tells us to differentiate the function inside the brackets and the letter being used to differentiate with respect to is x. In other words, differentiate the $x's$ in the brackets. $\frac{d}{dx}$ is an operator that means "take the derivative with respect to x" whereas $\frac{dy}{dx}$ indicates to "take the derivative of y with respect to x". We could write $\frac{d}{dx}(y)$ and it would mean the same as $\frac{dy}{dx}$. $= 6x^2 + 8x - 3$	**Find the rate of change of** $\boldsymbol{y = 2x^3 + 4x^2 - 3x + 1}$ **with respect to** $\boldsymbol{x}$ The rate of change is just the gradient. rate of change $= 6x^2 + 8x - 3$ The rate of change is $\frac{dy}{dx}$, so we should have just written $\frac{dy}{dx}$. $\frac{dy}{dx} = 6x^2 + 8x - 3$

Notice how the answer for all above is $6x^2+8x-3$. The only thing that changes is the green part

Let's look at the second derivative

$y=2x^3+4x^2-3x+1$ **Find** $\frac{d^2y}{dx^2}$	$f(x)=2x^3+4x^2-3x+1$ **Find** $f''(x)$	$\frac{d^2}{dx^2}(2x^3+4x^2-3x+10)$	**Find the <u>rate of</u> the rate of change of** $y=2x^3+4x^2-3x+1$ **with respect to** x Here we want the RATE OF the rate i.e. the rate of change of the gradient.
$\frac{d^2y}{dx^2}$ means differentiate the function y TWICE $\frac{dy}{dx}=12x+8$	$f'(x)=12x+8$	$=12x+8$	$\frac{dy}{dx}=12x+8$

You should now understand that the following are equivalent ways of representing being told to differentiate (find …) or that we have differentiated:

$$\frac{dy}{dx} \text{ or } f'(x) \text{ or } y'$$

$\frac{dy}{dx}$ means the **rate of change of** y with **respect to** x

- If given y we use the notation y' or $\frac{dy}{dx}$
- If given $f(x)$ we use the notation $f'(x)$
- If we differentiate twice we write $\frac{d^2y}{dx^2}$ or $f''(x)$ or y''
- The general formula if we want to differentiate n times is $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$ or $y^{(n)}$

For example, if we differentiate a function 5 times we would write $\frac{d^5y}{dx^5}$ or $f'''''(x)$ or y'''''. Instead of writing $f'''''(x)$ or y''''' it is better to write $f^{(5)}(x)$ or $y^{(5)}$.

We can use a fancy notation to represent the derivative a point specific $x=b$ (meaning we have plugged the derivative in and gotten a value). We can write $\left.\frac{d}{dx}\right|_{x=b}$ or $f'(b)$

For example, find the derivative of $y=x^2-4x$ at $x=3$.

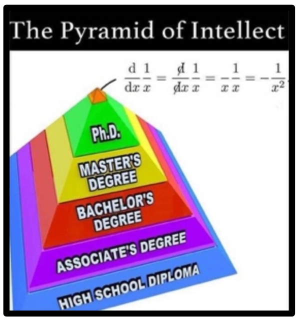
$$\frac{dy}{dx}=2x-4$$

$$\frac{dy}{dx}=2(3)-4=2$$

We can write $\left.\frac{d}{dx}\right|_{x=3}=2$

We can also $f'(3)=2$ which just says the derivative/gradient/slope at $x=3$ is 2.

4 Dealing With Harder Types Of Questions



What about if we have multiplication or division of terms (fractions) or roots? Such as

$$(x-2)(x+3) \text{ or } \frac{5}{x} \text{ or } 2\sqrt{x}$$

We cannot just differentiate straight away like we did when we had a sum or difference of single terms like

$$2x^3 + 4x^2 - 3x + 1$$

Why can't we? The above was easy as we could quickly deal with each term one by one and apply the rule

"bring the power down to multiply with the coefficient at the front and take 1 away from the power".

But how do we apply this rule to brackets, roots or fractions? We can't! We can only apply this rule to a sum or difference of single terms **WITHOUT brackets, roots or fractions**. We want all variables (letters) to have powers and there to be NO LETTERS in the denominators. **These harder questions are all about getting the question in the correct form to differentiate i.e. a sum or difference of single terms WITHOUT brackets, roots or fractions where the variables have powers.** Only then can we apply the rule

"bring the power down to multiply with the coefficient at the front and take 1 away from the power".

This means we must simplify first by either:

- **Getting rid of roots :** we use the rational power indices rules

$$\sqrt{x} = x^{\frac{1}{2}}$$
$$\sqrt[3]{x} = x^{\frac{1}{3}}$$
$$\sqrt[b]{x^a} = x^{\frac{a}{b}}$$

- **Getting rid of brackets** : we expand brackets. We will often need to using indices rules for multiplication

$$x^a \times x^b = x^{a+b}$$

- **Getting rid of fractions** : we either use
    - Division indices rule: $x^a \div x^b$ or $\frac{x^a}{x^b} = x^{a-b}$
    - Negative power indices rule : $\frac{a}{x^n} = ax^{-n}$ or $\frac{a}{bx^n} = \frac{a}{b}x^{-n}$
    - Split up fractions. This is just the reverse of finding a common denominator. You will be familiar with this rule from right to left when adding fractions, but here we are using it from left to right i.e. doing the opposite.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

Then use negative power indices rule on both fractions: $\frac{a}{x^n} = ax^{-n}$ or $\frac{a}{bx^n} = \frac{a}{b}x^{-n}$

You must know your indices rules well in order to be good at these types of harder questions! If you do not, go back and learn them now! The following are examples and methods of all types that could come up and if you understand all then you will be in great shape.

Types and Method

Brackets

We expand them

$$(x+2)(x-4)$$

$$\boxed{x^2 - 4x + 2x - 8}$$

Don't mistake the common mistake of thinking: $(2x)^2$ is the same as $(2+x)^2$

$$(2x)^2 = 4x^2$$
Whereas
$$(2+x)^2 = 4 + 4x + x^2$$

The latter requires expanding the brackets since there is a sign ($+$ or $-$) in the middle.

Multiplication and Division Indices Rules:

$$x^a \times x^b = x^{a+b}$$
$$\frac{x^a}{x^b} = x^{a-b}$$

Multiplying:

Add the powers
$x^a \times x^b = x^{a+b}$
The bases must be the same to use this rule and notice how they do not change (x stays x)

Dividing:

Subtract the powers
$x^a \div x^b = x^{a-b}$
The bases must be the same to use this rule and notice how they do not change (x stays x)

Negative Power Indices Rules:

$\frac{3}{x}$ means $3x^{-1}$

$\frac{3}{5x}$ means $\frac{3}{5}x^{-1}$. We don't need to do anything about the 5. Only the variable part needs to be index form

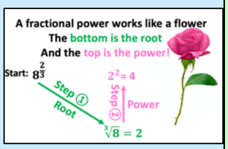
$$\frac{a}{bx^n} = \frac{a}{b}x^{-n}$$

The easiest way to think of this rule is that if we move terms between the numerator and denominator, the POWER of what is being moved changes/reverses its sign (a positive becomes a negative and vice versa). Notice how the coefficients don't change.

****IMPORTANT****
It is **ok to have numbers in the denominators for fractions**, but not variables
$\frac{x}{2}$ means $\frac{1}{2}x$ and is in a good form to differentiate without using the negative power rule.

Radicals (Roots) Indices Rules:

$$\sqrt{x} = x^{\frac{1}{2}}$$
$$\sqrt[3]{x} = x^{\frac{1}{3}}$$
$$\sqrt[b]{x^a} = x^{\frac{a}{b}}$$



Splitting Up Fractions:

Way 1: Best Method
$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$
Then use $\frac{x^a}{x^b} = x^{a-b}$ for each fraction

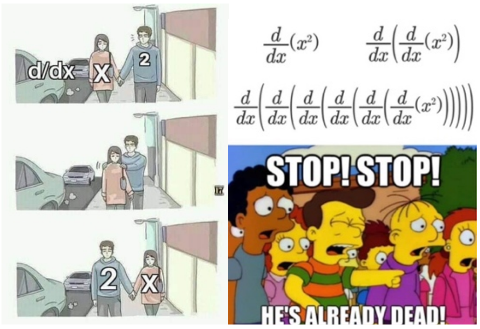
Way 2:
you can also write $\frac{a+b}{c}$ as $(a+b)c^{-1}$ and multiply out using $x^a \times x^b = x^{a+b}$ rule.

Do not try and simplify $\frac{a}{b+c}$. The only thing you can do is bring the whole of b+c up so we get $a(b+c)^{-1}$, but this is not helpful. You won't get types like these.

Examples	$y = x(x+4)(x+2)$ Simplify first $y = x(x^2+6x+8)$ Remember that we add the powers when multiplying $x^a \times x^b = x^{a+b}$ $y = x^{1+2} + 6x^{1+1} + 8x$ $y = x^3 + 6x^2 + 8x$ This is now in the correct form to differentiate $\frac{dy}{dx} = 3x^2 + 12x$	$y = \sqrt{x}(3x - 4x^2)$ Simplify first $y = x^{\frac{1}{2}}(3x - 4x^2)$ Remember that we add the powers when multiplying $x^a \times x^b = x^{a+b}$ $y = 3x^{1+\frac{1}{2}} - 4x^{2+\frac{1}{2}}$ $y = 3x^{\frac{3}{2}} - 4x^{\frac{5}{2}}$ This is now in the correct form to differentiate $\frac{dy}{dx} = \frac{9}{2}x^{\frac{1}{2}} - 10x^{\frac{3}{2}}$	$y = \frac{2}{5x} + \frac{x}{2}$ Simplify first using negative powers rule $y = \frac{2}{5x^1} + \frac{1x}{2}$ $y = \frac{2}{5}x^{-1} + \frac{1}{2}x$ This is now in the correct form to differentiate $\frac{dy}{dx} = -\frac{2}{5}x^{-2} + \frac{1}{2}$	$y = 5\sqrt[3]{x^2}$ Simplify first using rational power indices rule $y = 5x^{\frac{2}{3}}$ This is now in the correct form to differentiate $\frac{dy}{dx} = \frac{10}{3}x^{-\frac{1}{3}}$	$y = \frac{(x+4)(x-1)}{3\sqrt{x}}$ Simplify first $y = \frac{x^2+3x-4}{3x^{\frac{1}{2}}}$ We split up into fractions $y = \frac{x^2}{3x^{\frac{1}{2}}} + \frac{3x}{3x^{\frac{1}{2}}} + \frac{-4}{3x^{\frac{1}{2}}}$ Now we use the division indices rule for the first 2 terms $x^a \div x^b = x^{a-b}$ And the negative power rule for the final term The constants just stay $y = \frac{x^2}{3x^{\frac{1}{2}}} + \frac{3x^1}{3x^{\frac{1}{2}}} + \frac{-4}{3x^{\frac{1}{2}}}$ $y = \frac{1}{3}x^{2-\frac{1}{2}} + 1x^{1-\frac{1}{2}} - \frac{4}{3}x^{-\frac{1}{2}}$ $y = \frac{1}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} - \frac{4}{3}x^{-\frac{1}{2}}$ This is now in the correct form to differentiate $\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{2}{3}x^{-\frac{3}{2}}$

You will learn another way to deal with products and fractions (quotients) later on in your course when you learn product and quotient rule, but for now with our limited differentiation knowledge we want to get rid of them.

5 Summary Table



You need to make sure you have read all of this document before looking at this table.

For all general types of functions the differentiation rules looks like:

Common Functions	Function	Derivative
Constant	c	0
Line	x ax	1 a
Square	ax^2	$2ax$
Cubic	ax^3	$3ax^2$
Root	$a\sqrt{x}$ We need to use indices rules on this This is the same as writing $ax^{\frac{1}{2}}$	$\frac{1}{2}ax^{-\frac{1}{2}}$
Reciprocal	$\frac{a}{x}$ We need to use indices rules on this This is the same as writing ax^{-1}	$\frac{-a}{x^2}$
General Power	ax^n	anx^{n-1}

This can be written formerly as:

- We differentiate term by term when we have a sum or difference
$$\frac{d}{dx}(f(x) \pm\ g(x)) = \frac{d}{dx}f(x) \pm\ \frac{d}{dx}g(x)$$

- We can take the constant out and it is just the constant times the derivative of the function)
$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$$

- Constants go to zero when differentiated
$$\frac{d}{dx}(c) = 0$$

- The rule to differentiate is bring the power down and multiply it with whatever is there and take one off the power
$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Note: You won't learn how to deal with products $\frac{d}{dx}\big(f(x)g(x)\big)$ and quotients $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ until year 2.